

# Concrete / Pictorial / Abstract

## Maths Calculation Policy

This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added. It is a working document and will be revised and amended as necessary. Many variations have been included to provide teachers with a range of tools to support pupils in their grasp of number and calculation. To ensure consistency for pupils, it is important that the mathematical language used in maths lessons reflects the vocabulary used throughout this policy.



## Recommended practice delivering a mastery approach

True mastery aims to develop all children's mathematical understanding at the same pace. As much as possible, children should be accessing the same learning. Differentiation should primarily be through support, scaffolding and deepening, not through task.

Consistency in language is essential for pupils to understand the concepts presented in mathematics. If other, 'child-friendly' terminology is used, this must be alongside the current terminology recommended by maths specialists. Using this will support children with their examinations and throughout secondary school.

Evidence repeatedly shows that mixed ability seating increases less confident pupils' perception of mathematical capability, which impacts positively upon outcomes. While not a school policy, it is recommended to avoid ability groups. This presents a challenge in ensuring the more confident mathematicians are being extended. An extension tasks to deepen understanding is the most simplistic way around this.

Concrete, pictorial, abstract (CPA) concepts should not be confused as differentiation for lower, middle, higher attaining children. CPA is an approach to be used with the whole class and teachers should promote each area as equally valid. Manipulatives in particular must not be presented as a resource to support the less confident or lower attaining pupils.



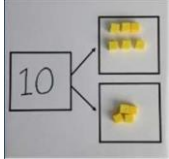

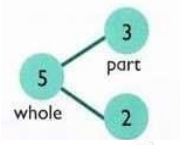

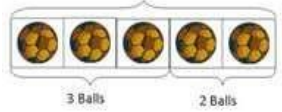


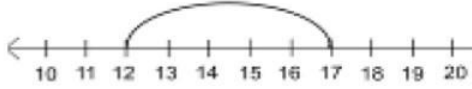
Used well, manipulatives can enable pupils to inquire themselves- becoming independent learners and thinkers. They can also provide a common language with which to communicate cognitive models for abstract ideas. Drury, H. (2015)

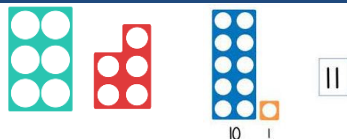
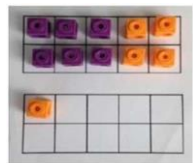
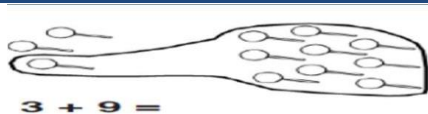
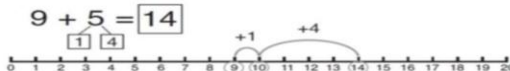

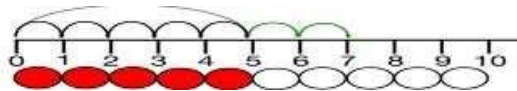
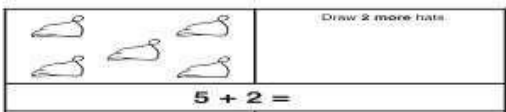
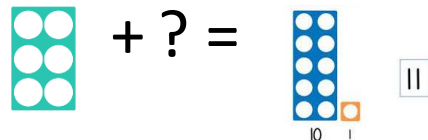
Children aged seven to ten years old work in primarily concrete ways and that the abstract notions of mathematics may only be accessible to them through embodiment in practical resources. Jean Piaget's (1951)

Real things and structured images enables children to understand the abstract. The concrete and the images are a means for children to understand the symbolic so it's important to move between all modes to allow children to make connections. Morgan, D. (2016)

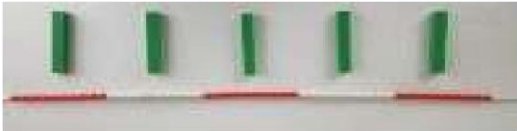
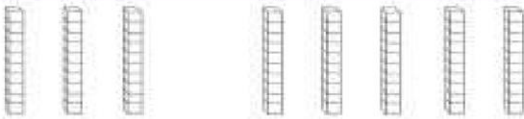
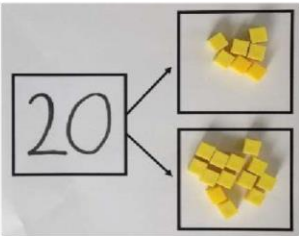
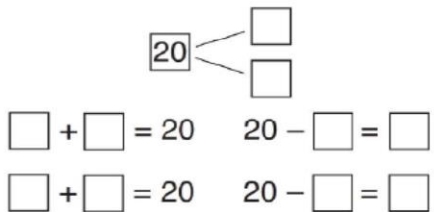
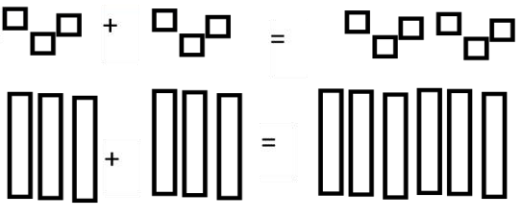
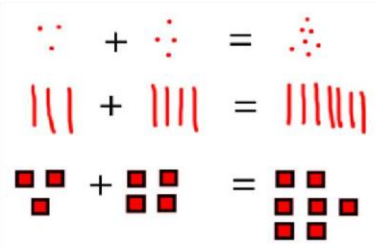
The abstract should run alongside the concrete and pictorial stage as this enables pupils to better understand mathematical statements and concept

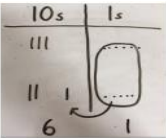

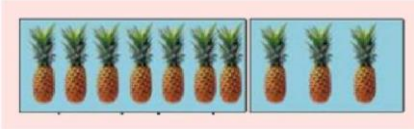
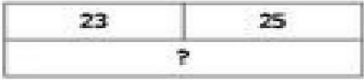
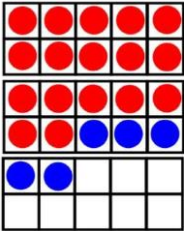
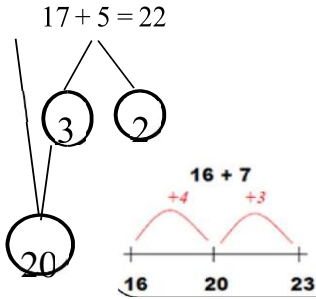
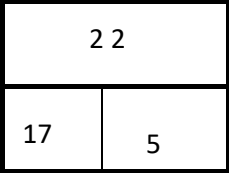
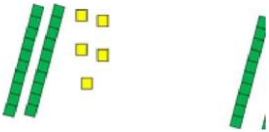
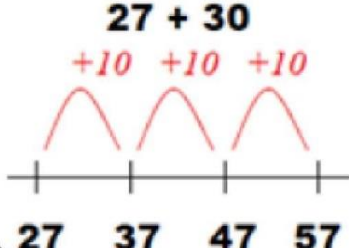
# YEAR 1 Addition

Objective / Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	 <p>Use part, part whole model.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p>  	    <p>Use pictures to add two numbers together as a group or in a bar.</p>	$8 = 5 + 3$ $5 + 3 = 8$  <p>Use the part, part whole diagram as shown above to move into the abstract.</p> <p>Include missing number questions to support varied fluency:</p> $8 = ? + 3$ $5 + ? = 8$
Starting at the bigger number and counting on	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	 $12 + 5 = 17$ <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	$5 + 12 = 17$ <p>Place the larger number in your head and count on the smaller number to find your answer.</p>

<p>Regrouping to make 10.</p> <p>This is an essential skill for column addition later.</p>	<div><div></div><div></div></div> <p>Start with the bigger number and use the smaller number to make 10. Use ten frames.</p>	<div><div><p>6 + 5 = 11</p><p>3 + 9 =</p></div><div><p>Use pictures or a number line. Regroup or partition the smaller number using the part, part whole model to make 10.</p></div><div><p>Children to draw the ten frame and use images of Numicon, counters or cubes to help.</p><table border="1" data-bbox="940 499 1202 580"><tr><td>x</td><td>x</td><td>x</td><td>x</td><td>x</td></tr><tr><td>x</td><td>o</td><td>o</td><td>o</td><td>o</td></tr></table><div><table border="1" data-bbox="940 619 1209 702"><tr><td>o</td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr></table><p>_____ = 6 + 5</p></div></div></div>	x	x	x	x	x	x	o	o	o	o	o										<div><p>7 + 4 = ?</p><p>7 + 4 = 11</p><p>If I am at seven, how many more do I need to make 10? How many more do I have left to add on now?</p></div>
x	x	x	x	x																			
x	o	o	o	o																			
o																							
<p>Represent &amp; use number bonds and related subtraction facts within 20</p>	<div><div></div><p>2 more than 5.</p></div>	<div><div></div><div></div></div>	<div><p>Include missing number questions:</p><p>8 = ? + 3</p><p>5 + ? = 8</p><p>Emphasis should be on the language</p><p>‘1 more than 5 is equal to 6.’</p><p>‘2 more than 5 is 7.’</p><p>‘8 is 3 more than 5.’</p></div>																				
<p>Children to begin to develop an understanding of equality</p>	<div><div></div></div>	<div><table border="1" data-bbox="1093 1117 1411 1420"><tr><td>6</td><td>?</td></tr><tr><td colspan="2">11</td></tr></table></div>	6	?	11		<div><p>6 + ____ = 11</p></div>																
6	?																						
11																							

# YEAR 2 Addition

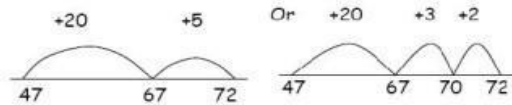
Objective /Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	<p>50= 30 + 20</p>  <p>Model using dienes and bead strings</p>	 <p>3 tens + 5 tens = _____ tens 30 + 50 = _____</p> <p>Use representations for base ten.</p>	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$
Use known number facts <b>Part, part whole</b>	 <p>Children explore ways of making numbers within 20</p>	 <p> <math>\square + \square = 20</math>    <math>20 - \square = \square</math>  <math>\square + \square = 20</math>    <math>20 - \square = \square</math> </p>	<p>Explore commutativity of addition by swapping the addends to build a fact family. Explore the concept of the inverse relationship of addition and subtractions and use this to check calculations.</p> <p> <math>\square + 1 = 16</math>    <math>16 - 1 = \square</math>  <math>1 + \square = 16</math>    <math>16 - \square = 1</math> </p>
Using known facts		 <p>Children draw representations of H,T and O</p>	<p> <math>3 + 4 = 7</math>  <i>leads to</i>  <math>30 + 40 = 70</math>  <i>leads to</i>  <math>300 + 400 = 700</math> </p>

		<p>Also, use in a place value chart, using Diennes or place value counters.</p> 	
Bar model	 $3 + 4 = 7$	 $7 + 3 = 10$	 $23 + 25 = 48$
Add a two digit number and ones	 <p>17 + 5 = 22 Use ten frame</p> <p>Children explore the pattern. 17 + 5 = 22 27 + 5 = 32</p>	<p>17 + 5 = 22</p> <p>Use part part whole and number line to model</p> 	<p>17 + 5 = 22</p> <p>Explore related facts</p> <p>17 + 5 = 22</p> <p>5 + 17 = 22</p> <p>22 - 17 = 5</p> <p>22 - 5 = 17</p> 
Add a 2 digit number and tens	 $25 + 10 = 35$ <p>Explore that the ones digit does not change</p>	$27 + 30$ 	<p>27 + 10 = 37</p> <p>27 + 20 = 47</p> <p>27 + □ = 57</p>

Add two 2-digit numbers



Model using dienes , place value counters and Numicon



Use number line and bridge ten using part whole if necessary.

$$\begin{array}{r} 25 + 47 \\ \swarrow \quad \searrow \\ 20 + 5 \quad 40 + 7 \end{array}$$

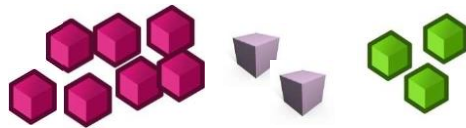
$$20 + 40 = 60$$

$$5 + 7 = 12$$

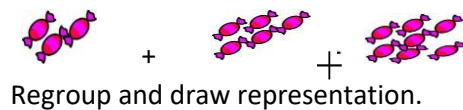
$$60 + 12 = 72$$

**Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values.**

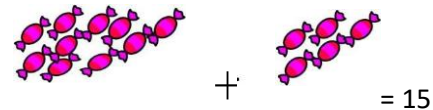
Add three 1-digit numbers



Combine to make 10 first if possible, or bridge 10 then add third digit



Regroup and draw representation.



$$= 15$$

$$\begin{array}{r} (4) + 7 + (6) = \boxed{10} + \boxed{7} \\ 10 \\ = \boxed{17} \end{array}$$

Combine the two numbers that make/ bridge ten then add on the third.

Children to begin to develop an understanding of equality

$$6 + 5 = 5 + \underline{\quad}$$

$$6 + 5 = \underline{\quad} + 4$$

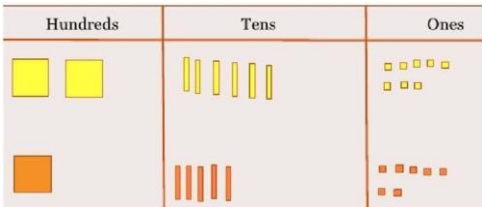
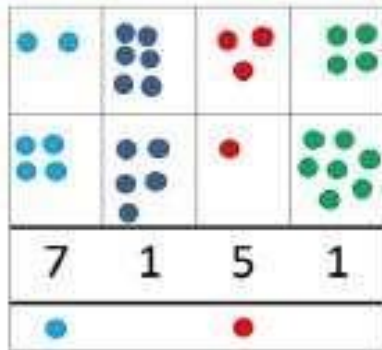
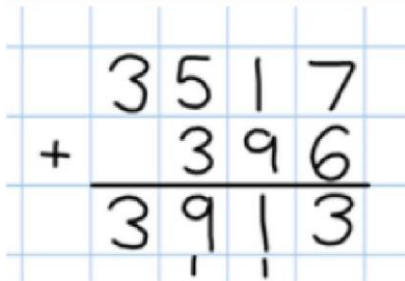
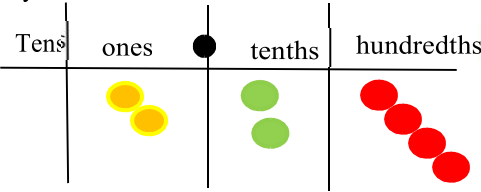
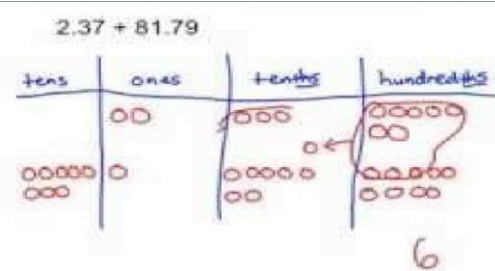
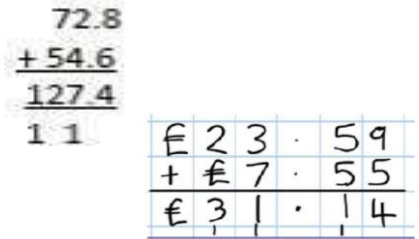
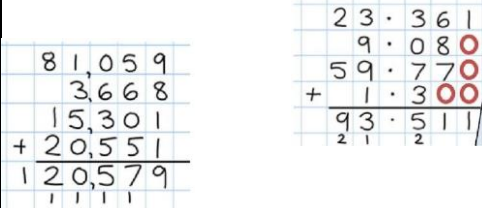
# YEAR 3 Addition

Objective /Strategy	Concrete	Pictorial	Abstract
<p>Column Addition—no regrouping (friendly numbers)</p> <p>Add two or three 2 or 3digit numbers.</p>	<div><div><div>T</div><div>O</div></div><div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div></div><div>Dienes or Numicon</div><div>Add together the ones first, then the tens.</div><div><div><div>Tens</div><div>Units</div></div><div><div><div>45</div><div>34</div></div><div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div></div><div><div>7</div><div>9</div></div></div><div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div></div><div>Calculations</div><div><div>21 + 42 =</div><div>21</div><div>+ 42</div></div><div>Move to using place value counters</div></div></div>	<p>Children move to drawing the counters in a place value chart.</p> <div><div>tens</div><div>ones</div></div> <div><div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div></div>	<div><div>223 +</div><div>114</div><div>337</div></div> <div>Add the ones first, then the tens, then the hundreds.</div>
<p>Column Addition with regrouping.</p>	<div><div><div>Tens</div><div>Units</div></div><div><div><div>39</div><div>15</div></div><div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div></div><div><div>5</div><div>4</div></div></div></div> <div>Exchange ten ones for a ten. Model using Numicon and place value counters.</div>	<div><div><div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div></div><div>34</div><div>+17</div><div><div>5</div><div>1</div></div></div>	<div><div>20 + 5</div><div>40 + 8</div><div>60 + 13 = 73</div></div> <div>Start by partitioning the numbers before formal column to show the exchange.</div> <div><div>536</div><div>+ 85</div><div>621</div><div>11</div></div>

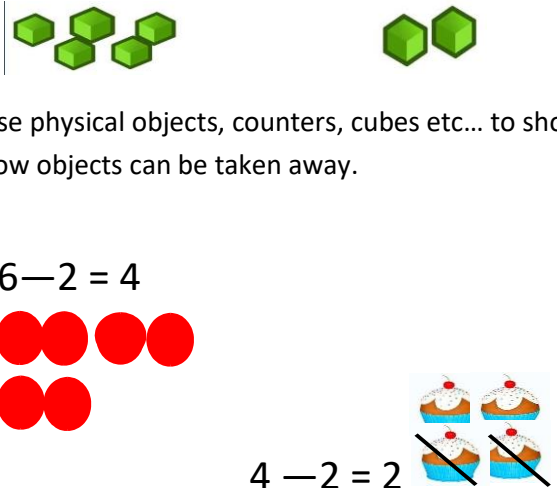
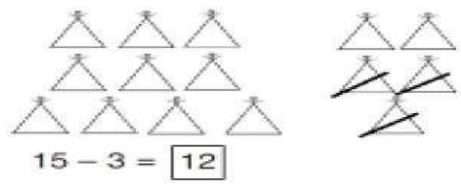

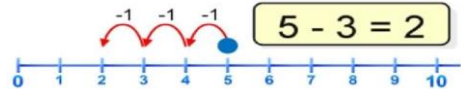


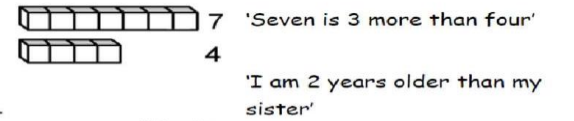
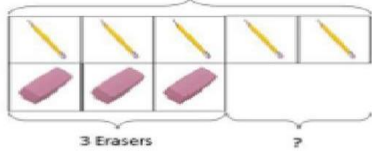
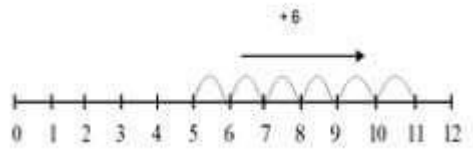
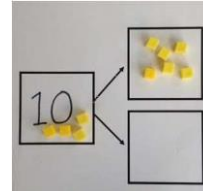
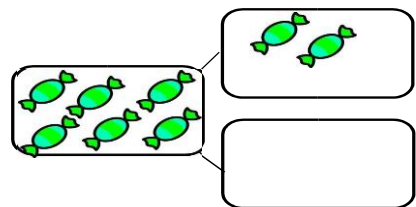
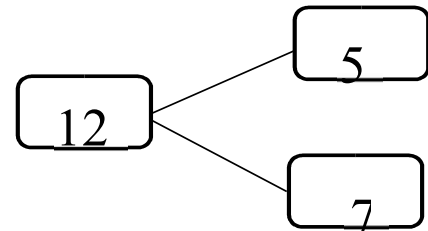
	<div data-bbox="622 97 853 209"></div> <div data-bbox="481 229 784 280"><math>46 + 27 = 73</math></div>		
<div data-bbox="98 304 362 451">Estimate the answers to questions and use inverse operations to check answers</div>	<div data-bbox="380 304 792 411"></div> <div data-bbox="394 435 678 469">Estimating <math>98 + 17 = ?</math></div> <div data-bbox="394 499 584 531"><math>100 + 20 = 120</math></div>	<div data-bbox="920 304 1413 335">Use number lines to illustrate estimation.</div> <div data-bbox="931 422 1350 499"></div>	<div data-bbox="1570 304 2085 376">Building up known facts and using them to illustrate the inverse and to check answers:</div> <div data-bbox="1563 422 2016 453"><math>98 + 18 = 116</math>      <math>116 - 18 = 98</math></div> <div data-bbox="1563 499 2016 531"><math>18 + 98 = 116</math>      <math>116 - 98 = 18</math></div>

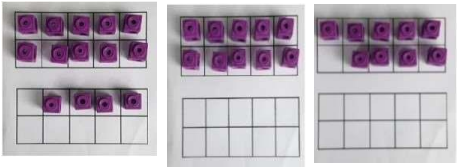
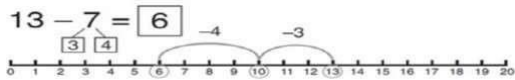
# YEARS 4 – 6 Addition

Objective /Strategy	Concrete	Pictorial	Abstract
Years 4 – 6 Estimate and use inverse operations to check answers to a calculation	As per Year 3		
Y4—add numbers with up to 4 digits	<p>Children continue to use Dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.</p> 	 <p>Draw representations using place value grid.</p>	 <p>Continue from previous work to carry hundreds as well as tens.</p> <p>Relate to money and measures.</p>
Y5—add numbers with more than 4 digits.  Add decimals with 2 decimal places, including money.	<p>As year 4</p>  <p>Introduce decimal place value counters and model exchange for addition.</p>		
Y6—add several numbers of increasing complexity, including adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5	<p>Insert zeros for place holders.</p> 

# YEAR 1 SUBTRACTION

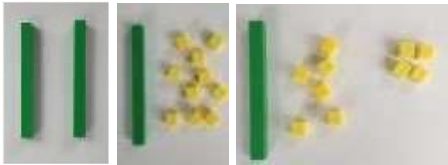
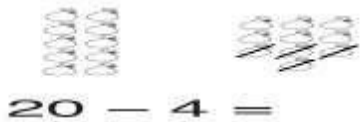
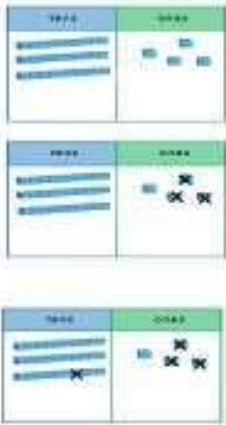
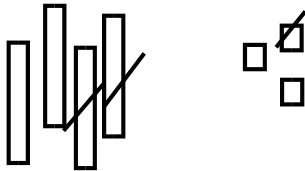
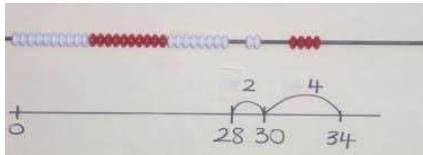
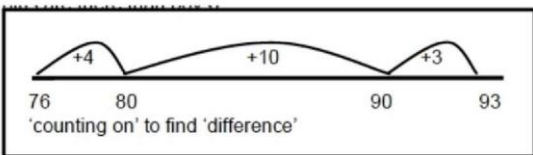
Objective /Strategy	Concrete	Pictorial	Abstract
Subtracting ones	 <p>Use physical objects, counters, cubes etc... to show how objects can be taken away.</p> <p><math>6 - 2 = 4</math></p> <p><math>4 - 2 = 2</math></p>	<p>Cross out drawn objects to show what has been removed.</p>  <p><math>15 - 3 = 12</math></p>	<p><math>7 - 4 = 3</math></p> <p><math>16 - 9 = 7</math></p>
Counting back	<p>Move objects away from the group, counting backwards.</p>  <p>Move the beads along the bead string as you count backwards.</p>	 <p><math>5 - 3 = 2</math></p> <p>Count back in ones using a number line.</p>	<p>Put 13 in your head, count back 4.</p> <p>What number are you at?</p>

<p>Find the Difference</p>	<p>Compare objects and amounts</p>  <p>5 Pencils</p>  <p>3 Erasers</p> <p>Lay objects to represent bar model.</p>	<p>Count on using a number line to find the difference.</p>  <p>Use Numicon laid on top of one another to show the difference.</p>	<p>Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister.?</p> <p>Children to explore why <math>9 - 6</math>, <math>8 - 5</math>, <math>7 - 4</math> have the same difference.</p>
<p>Represent and use number bonds and related subtraction facts within 20</p> <p>Include subtracting zero</p> <p>Part, Part Whole model</p>	 <p>Link to addition. Use PPW model to model the inverse.</p> <p>If 10 is the whole and 6 is one of the parts, what is the other part?</p> $10 - 6 = 4$		<p>Move to using numbers within the part whole model.</p>  <p>Include missing number problems:</p> $12 - ? = 5$ $12 - ? = 7$

Objective/Strategy	Concrete	Pictorial	Abstract
<b>Make 10</b>	<p>14—9</p>  <p>Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.</p>	<p>13—7 = 6</p>  <p>13—7</p> <p>Jump back 3 first, then another 4. Use ten as the stopping point.</p>	<p>16—8</p> <p>How many do we take off first to get to 10? How many left to take off?</p> <p>Children to show how they can make 10 by partitioning then subtracting</p> <p>14—5 =</p> <p style="margin-left: 40px;">/ \</p> <p style="margin-left: 40px;">4 1</p> <p>14—4 = 10</p> <p>10—1 = 9</p>

Bar model			<table border="1" data-bbox="1563 882 2036 981"><tr><td>8</td><td>2</td></tr></table>	8	2
8	2				
Including the inverse operations.	$5 - 2 = 3$		$10 = 8 + 2$ $10 = 2 + 8$ $10 - 2 = 8$ $10 - 8 = 2$		

# YEAR 2 - SUBTRACTION

Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 <p>Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'</p>	 $20 - 4 = 16$	$20 - 4 = 16$
Partitioning to subtract without regrouping. 'Friendly numbers'	$34 - 13 = 21$  <p>Use Dienes to show how to partition the number when subtracting without regrouping.</p>	<p>Children draw representations of Dienes and cross off.</p>  $43 - 21 = 22$	$43 - 21 = 22$ <p>Formal method?</p>
Make ten strategy Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.	 $34 - 28$ <p>Use a bead bar or bead strings to model counting to next ten and the rest.</p>	 <p>Use a number line to count on to next ten and then the rest.</p>	$93 - 76 = 17$

# YEAR 3 - SUBTRACTION

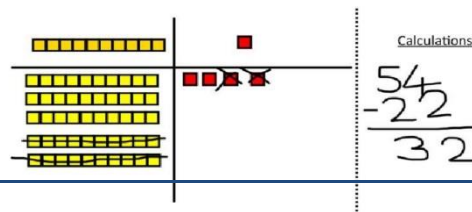
Objective/ Strategy

Concrete

Pictorial

Abstract

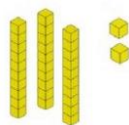
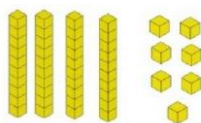
Subtract numbers mentally, including:  
three digit number + ones  
three digit number + tens  
three digit number + hundreds



Vary the position of the answer and question.  
Expose children to missing number questions and vary the missing part of the calculation.

$$\begin{aligned} 678 &= ? - 1 \\ 688 - 10 &= ? \\ 678 &= ? - 100 \end{aligned}$$

Column subtraction without regrouping (friendly numbers)



$$47 - 32$$

Use base 10 or Numicon to model

Draw representations to support understanding, e.g.  $34 - 12 = 22$

T | Ones

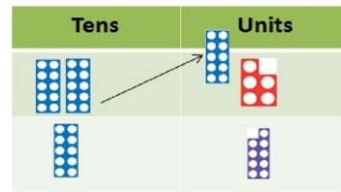
0 0 0 | 0 0 0 0

$$\begin{aligned} 47 - 24 &= 23 \\ \begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array} \end{aligned}$$

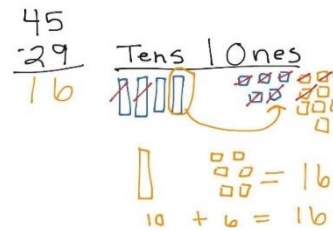
Intermediate step be needed to lead clear subtraction understanding.

may to

Column subtraction  
with regrouping



Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.



Children may draw base ten or pv counters and cross off.

$$\begin{array}{r} 836 - 254 = 582 \\ \begin{array}{r} \text{H} \quad \text{T} \quad \text{U} \\ 800 \quad 30 \quad 6 \\ - 200 \quad 50 \quad 4 \\ \hline 500 \quad 80 \quad 2 \end{array} \end{array}$$

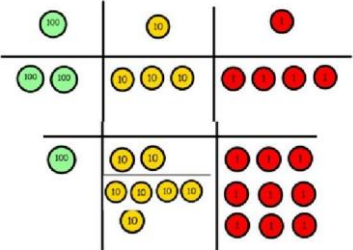
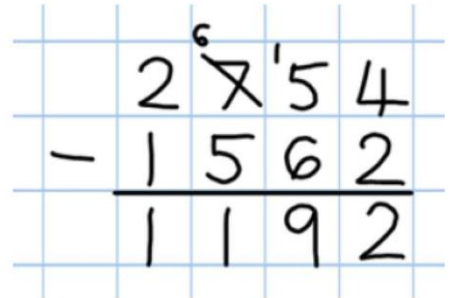
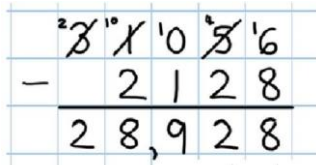
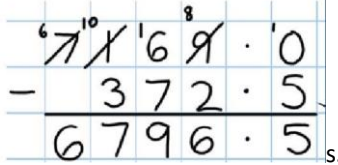
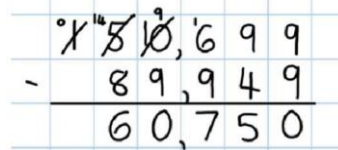
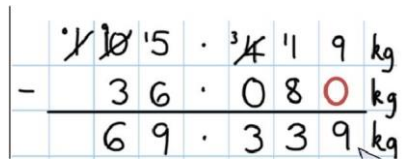
Begin by partitioning into pv columns.

$$\begin{array}{r} 728 - 582 = 146 \\ \begin{array}{r} \text{H} \quad \text{T} \quad \text{U} \\ 700 \quad 20 \quad 8 \\ - 500 \quad 80 \quad 2 \\ \hline 100 \quad 40 \quad 6 \end{array} \end{array}$$

Then move to formal method.

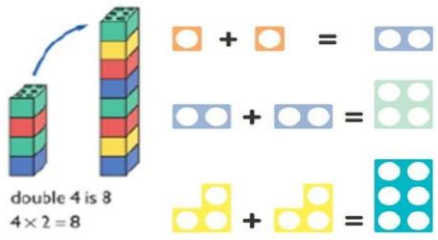

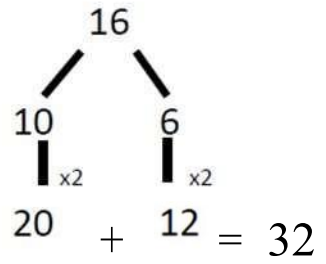
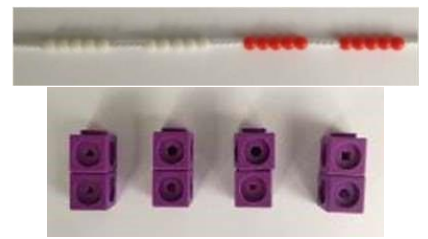
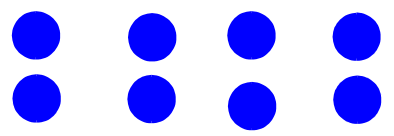
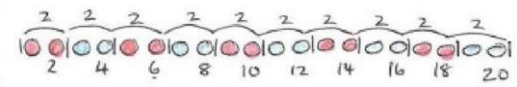


# YEARS 4 – 6 SUBTRACTION

Objective /Strategy	Concrete	Pictorial	Abstract
<p>Subtracting tens and ones</p> <p>Year 4 subtract with up to 4 digits.</p> <p>Introduce decimal subtraction through context of money</p>	<p>234 - 179</p>  <p>Model process of exchange using Numicon, base ten and then move to PV counters.</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	 <p>Use the phrase 'take and make' for exchange</p>
<p>Year 5- Subtract with at least 4 digits, including money and measures.</p> <p>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal</p> <p>Up to 3 decimal places</p>	<p>As Year 4</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	 <p>Use zeros for placeholder</p> 
<p>Year 6—Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal place).</p>	<p>As Year 4</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	 

# YEAR 1 MULTIPLICATION


Programme of Study specifies the following objectives, however it does not require the explicit teaching of the mathematical symbol of multiplication

Objective / Strategy	Concrete	Pictorial	Abstract
Doubling	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling</p> 	<p>Draw pictures to show how to double numbers</p> <p>Double 4 is 8</p> 	<p>Partition a number and then double each part before recombining it back together.</p> 
Counting in multiples (2s, 5s, 10s)	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting.</p> 	 <p>Children make representations to show counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>

Making equal groups and counting the total



Use manipulatives to create equal groups.

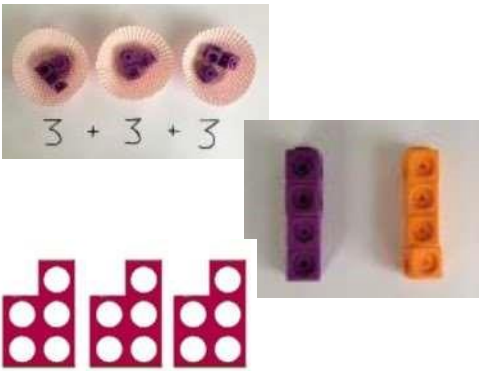
Draw  to show  $2 \times 3 = 6$

Draw and make representations

X X X	X X X	X X X
X X X	X X X	X X X

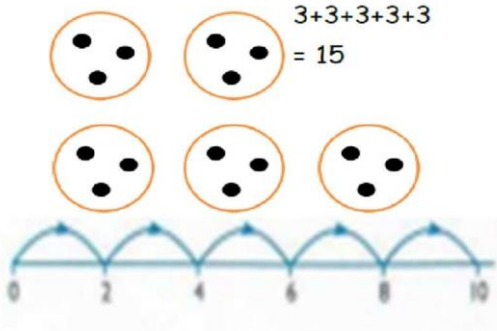
$$2 \times 4 = 8$$

Repeated addition



Use different objects to add equal groups

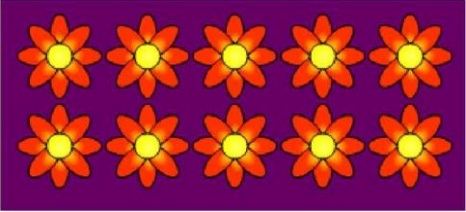
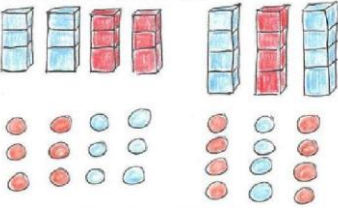
Use pictorial including number lines to solve prob There are 3 sweets in one bag.  
How many sweets are in 5 bags altogether?



Write addition sentences to describe objects and

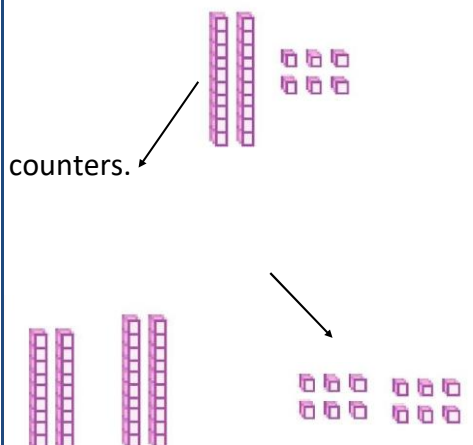
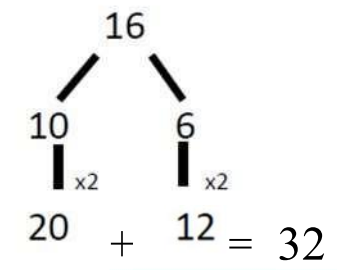


pictures.

Understanding arrays	<p>Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.</p> 	<p>Draw representations of arrays to show understanding</p> 	$3 \times 2 = 6$ $2 \times 5 = 10$
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# YEAR 2 MULTIPLICATION

Children should be able to recall and use multiplication and division facts for the 2, 5 and 10 times times tables.

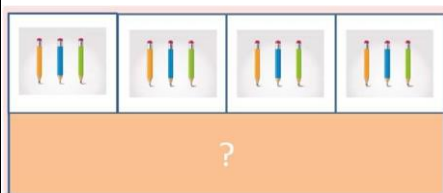
Objective / Strategy	Concrete	Pictorial	Abstract
Doubling	<p>Model doubling using dienes and PV</p>  <p>counters.</p> $40 + 12 = 52$	<p>Draw pictures and representations to show how to double numbers</p>	<p>Partition a number and then double each part before recombining it back together.</p>  $\begin{array}{c} 16 \\ \swarrow \quad \searrow \\ 10 \quad 6 \\ \downarrow \times 2 \quad \downarrow \times 2 \\ 20 \quad 12 \\ + \quad = 32 \end{array}$

Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)

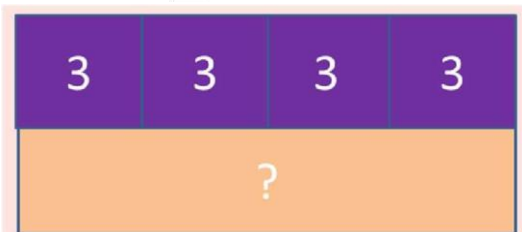
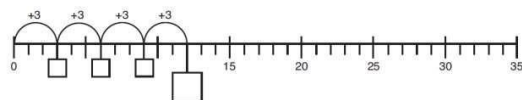
Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.



$$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$$



Number lines, counting sticks and bar models should be used to show representation of counting in multiples.



Count in multiples of a number aloud.

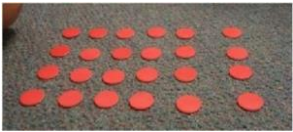



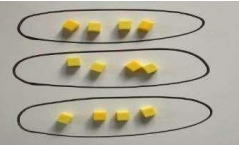
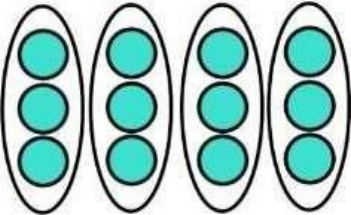
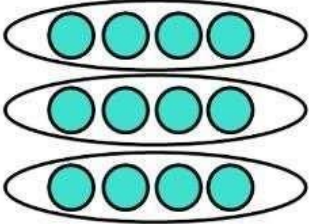

Write sequences with multiples of numbers.

0, 2, 4, 6, 8, 10

0, 3, 6, 9, 12, 15

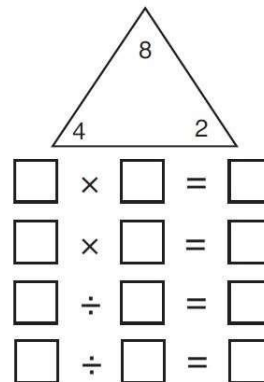
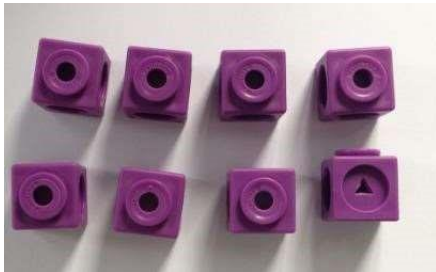
0, 5, 10, 15, 20, 25, 30

$$4 \times 3 = \square$$

Objective / Strategy	Concrete	Pictorial	Abstract
<p>Multiplication is commutative</p>	<p>Create arrays using counters and cubes and Numicon.</p>    <p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</p>  	<p>Use representations of arrays to show different calculations and explore commutativity.</p>  	<p><math>12 = 3 \times 4</math> <math>12 = 4 \times 3</math></p> <p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p><math>5 + 5 + 5 = 15</math>  <math>3 + 3 + 3 + 3 + 3 = 15</math>  <math>5 \times 3 = 15</math>  <math>3 \times 5 = 15</math></p>

## Using the Inverse

This should be taught alongside division, so pupils learn how they work alongside each other.



$$2 \times 4 = 8$$

$$4 \times 2 = 8$$

$$8 \div 2 = 4$$

$$8 \div 4 = 2$$

$$8 = 2 \times 4$$

$$8 = 4 \times 2$$

$$2 = 8 \div 4$$

$$4 = 8 \div 2$$

Show all 8 related fact family sentences.



# YEAR 3 MULTIPLICATION

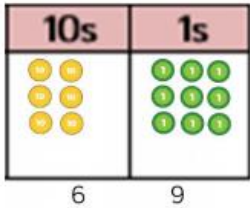
Children should be able to recall and use multiplication facts for the 3,4, and 8 times tables

Objective /Strateg	Concrete	Pictorial	Abstract
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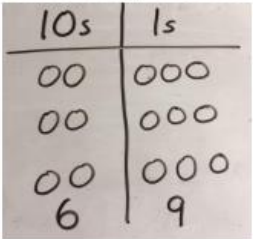
Progress to the formal method

Multiply 2 digit numbers by 1 digit numbers

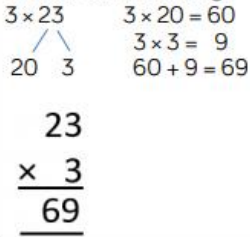
Formal column method with place value counters (base 10 can also be used.)  $3 \times 23$



Children to represent the counters pictorially.



Children to record what it is they are doing to show understanding.



Solve problems, including missing number problems, integer scaling problems,

Three times as high, eight times as long

$? \times 5 = 20$

$20 \div ? = 5$

3 hats and 4 coats, how many different outfits?

Conceptual variation; different ways to ask children to solve  $6 \times 23$

23

23

23

23

23

23

?

Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

With the counters, prove that  $6 \times 23 = 138$

Find the product of 6 and 23

$6 \times 23 =$   

$= 6 \times 23$

6

23

$\times$

23

—

—

What is the calculation? What is the product?

100s	10s	1s
	<div> <div></div> <div></div> <div></div> <div></div> <div></div> <div></div> </div>	<div> <div></div> <div></div> <div></div> <div></div> <div></div> <div></div> </div>

Conceptual variation; different ways to ask children to solve  $6 \times 23$

## YEARS 4 – 6 Multiplication

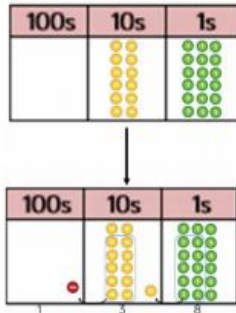
Objective /Strategy

Concrete

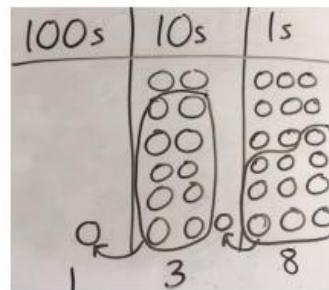
Pictorial

Abstract

**Formal column method** with place value counters.  
 $6 \times 23$



Children to represent the counters/base 10, pictorially  
 e.g. the image below.



Formal written method

$$\begin{array}{r}
 6 \times 23 = \\
 23 \\
 \times 6 \\
 \hline
 138 \\
 \hline
 11
 \end{array}$$

When children start to multiply  $3d \times 3d$  and  $4d \times 2d$  etc., they should be confident with the abstract:

To get 744 children have solved  $6 \times 124$ .  
 To get 2480 they have solved  $20 \times 124$ .

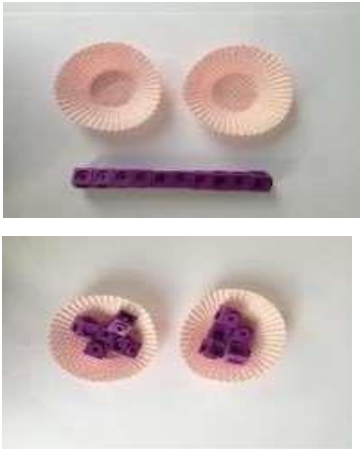
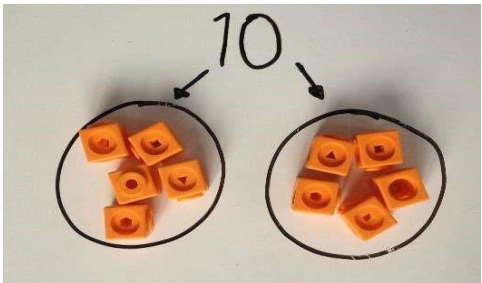

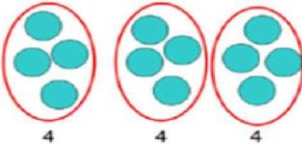
$$\begin{array}{r}
 124 \\
 \times 26 \\
 \hline
 744 \\
 2480 \\
 \hline
 3224
 \end{array}$$

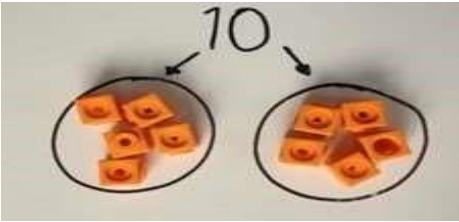
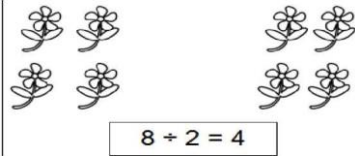
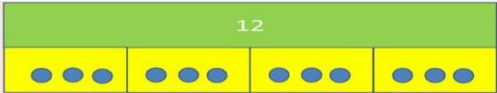
Answer: 3224

Objective/Strategy	Concrete	Pictorial	Abstract
<p>Multiplying decimals up to 2 decimal places by a single digit.</p>			<p>Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer.</p> 

# YEAR 1

Objective /Strategy	Concrete	Pictorial	Abstract
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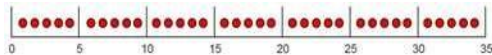
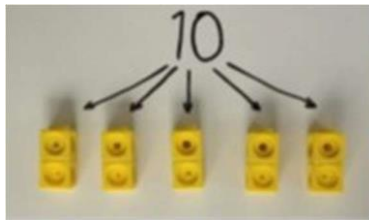
Objective/ Strategy	Concrete	Pictorial	Abstract
Division as sharing	  <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <p>8 shared between 2 is 4</p> <p>Sharing:</p>  <p>12 shared between 3 is 4</p>	<p>12 shared between 3 is 4</p>

Objective/Strategy	Concrete	Pictorial	Abstract
Division as sharing	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <p>Children use bar modelling to show and support understanding.</p>  <p><math>12 \div 4 = 3</math></p>	$12 \div 3 = 4$

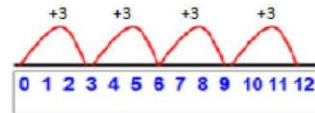
## Division as grouping

Divide quantities into equal groups.

Use cubes, counters, objects or place value counters to aid understanding.



Use number lines for grouping



$$12 \div 3 = 4$$

Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.



$$20 \div 5 = ?$$

$$5 \times ? = 20$$

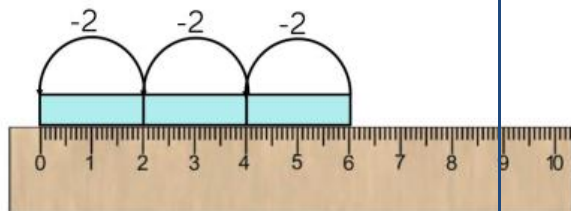
$$28 \div 7 = 4$$

Divide 28 into 7 groups. How many are in each group?



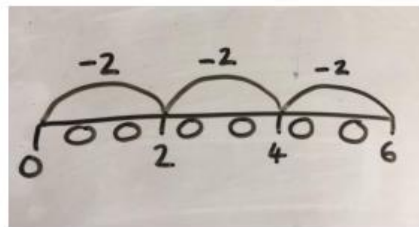
Repeated subtraction

Repeated subtraction using Cuisenaire rods above a ruler.  
 $6 \div 2$

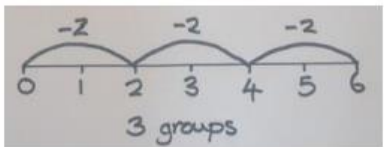


3 groups of 2

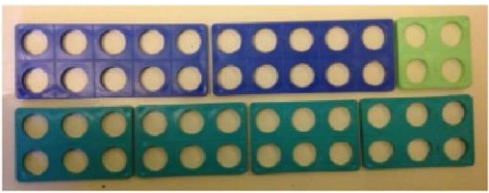
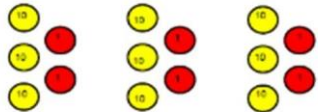
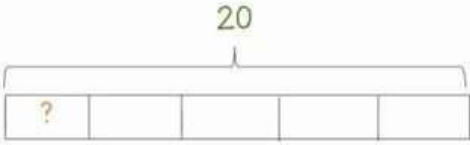
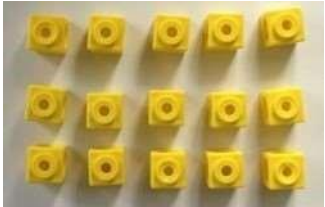
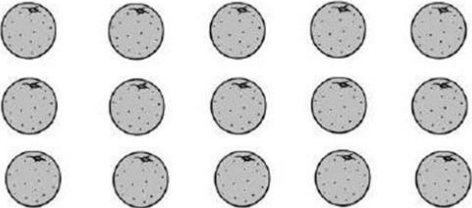
Children to represent repeated subtraction pictorially.



Abstract number line to represent the equal groups that have been subtracted.



# YEAR 2

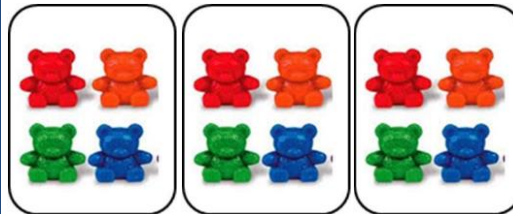
Objective/Strategy	Concrete	Pictorial	Abstract
Division as grouping	<p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> $96 \div 3 = 32$ 	<p>Continue to use bar modelling to aid solving division problems.</p>  $20 \div 5 = ?$ $5 \times ? = 20$	<p>How many groups of 6 in 24?</p> $24 \div 6 = 4$
Division with arrays	 <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg <math>15 \div 3 = 5</math>    <math>5 \times 3 = 15</math>  <math>15 \div 5 = 3</math>    <math>3 \times 5 = 15</math></p>	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences</p> 	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences. <math>7 \times 4 = 28</math></p> $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$

YEAR 3 <small>(Greater Depth Y2)</small>			
Objective/Strategy	Concrete	Pictorial	Abstract

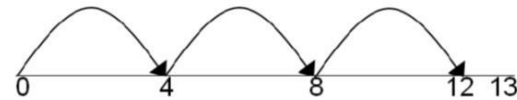
Division with remainders.

$$14 \div 3 =$$

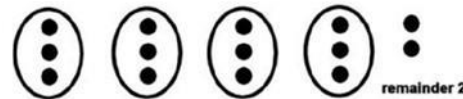
Divide objects between groups and see how much is left over



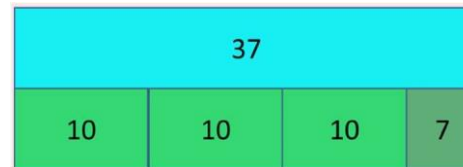
Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



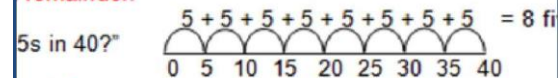
Draw dots and group them to divide an amount and clearly show a remainder.



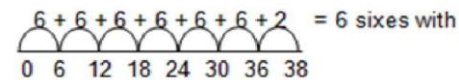
Use bar models to show division with remainders.



remainder:



remainder:



rs, when it becomes inefficient to count in single mu  
orded using known facts.

Complete written divisions and show the remainder using r.

$$29 \div 8 = 3 \text{ REMAINDER } 5$$

↑
↑
↑
↑

dividend
divisor
quotient
remainder

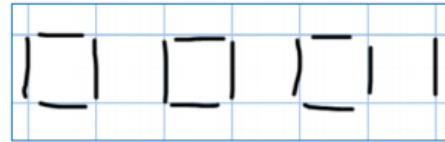
**2d + 1d with remainders** using lollipop sticks. Cuisenaire rods, above a ruler can also be used.  
 $13 \div 4$

Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.



There are 3 whole squares, with 1 left over.

Children to represent the lollipop sticks pictorially.

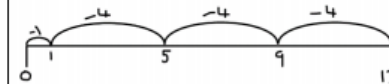


There are 3 whole squares, with 1 left over.

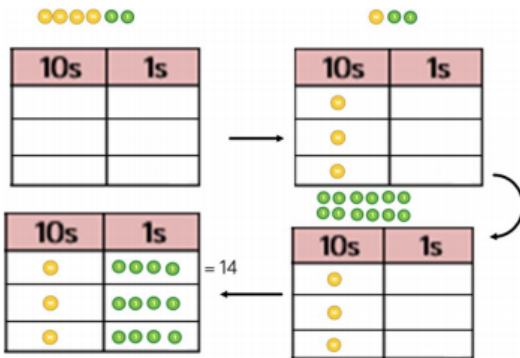
$13 \div 4 = 3$  remainder 1

Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

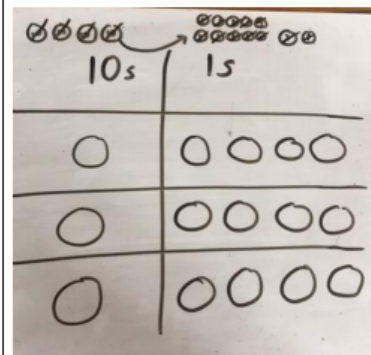
'3 groups of 4, with 1 left over'



**Sharing using place value counters.**  
 $42 \div 3 = 14$



Children to represent the place value counters pictorially.



Children to be able to make sense of the place value counters and write calculations to show the process.

$42 \div 3$   
 $42 = 30 + 12$   
 $30 \div 3 = 10$   
 $12 \div 3 = 4$   
 $10 + 4 = 14$

## Year 4-6

Objective/Strategy

Concrete

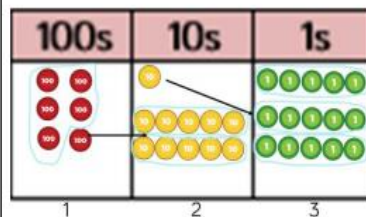
Pictorial

Abstract

Divide at least 3 digit numbers by 1 digit.

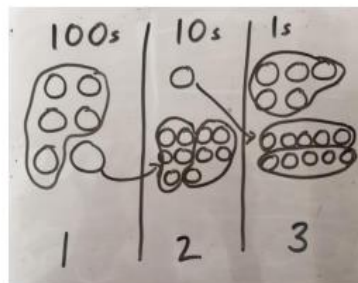
Short Division

**Short division** using place value counters to group.  
 $615 \div 5$



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.



Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 4 \overline{) 872} \\ \underline{8} \phantom{00} \\ 7 \phantom{00} \\ \underline{7} \phantom{00} \\ 2 \phantom{00} \\ \underline{2} \phantom{00} \\ 0 \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \\ \underline{4} \phantom{00} \\ 3 \phantom{00} \\ \underline{3} \phantom{00} \\ 2 \phantom{00} \\ \underline{0} \phantom{00} \\ 2 \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \\ \underline{35} \phantom{00} \\ 16 \phantom{00} \\ \underline{14} \phantom{00} \\ 21 \phantom{00} \\ \underline{21} \phantom{00} \\ 0 \end{array}$$

$$\begin{array}{r} 0663 \text{ r } 5 \\ 8 \overline{) 5309} \\ \underline{40} \phantom{00} \\ 13 \phantom{00} \\ \underline{8} \phantom{00} \\ 50 \phantom{00} \\ \underline{40} \phantom{00} \\ 10 \phantom{00} \\ \underline{8} \phantom{00} \\ 29 \phantom{00} \\ \underline{24} \phantom{00} \\ 5 \end{array}$$

# Long Division

Skill: Divide multi digits by 2-digits (short division)

Year: 6

		0	3	6
	12	4	4 <sub>3</sub>	7 <sub>2</sub>

$$432 \div 12 = 36$$

$$7,335 \div 15 = 489$$

	0	4	8	9
15	7	7 <sub>3</sub>	13 <sub>3</sub>	13 <sub>5</sub>

15	30	45	60	75	90	105	120	135	150
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When children begin to divide up to 4-digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.

# Skill: Divide multi digits by 2-digits (long division)

Year: 6

$$372 \div 15 = 24 \text{ r}12$$

			2	4	r	1	2
1	5	3	7	2			
	-	3	0	0			
			7	2			
	-		6	0			
			1	2			

- $1 \times 15 = 15$
- $2 \times 15 = 30$
- $3 \times 15 = 45$
- $4 \times 15 = 60$
- $5 \times 15 = 75$
- $10 \times 15 = 150$

When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction. This will depend on the context of the question.

Children can also answer questions where the quotient needs to be rounded according to the context.

			2	4	$\frac{4}{5}$
1	5	3	7	2	
	-	3	0	0	
			7	2	
	-		6	0	
			1	2	

$$372 \div 15 = 24\frac{4}{5}$$

Long Division